Tuning Communication Latency for Distributed MPC

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Distributed Control Systems

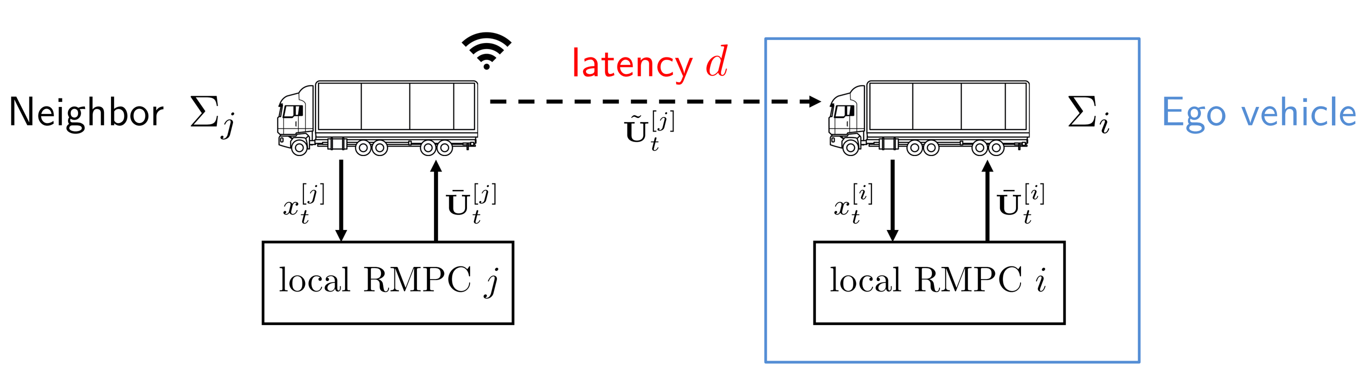
- Multi-agent control systems implemented over wireless communication networks, e.g. vehicle platoons, UAV formations, smart grids
  - ➢ Large scale networks
  - ➢ Fast and coupled dynamics
- Impact of communication latency
  - ➢ Degradation of control performance
  - ➢ Loss of safety guarantees

- **Research Objective**: Distributed control scheme for multi-agent systems that tunes communication latency to obtain better closed-loop control performance with safety guarantees

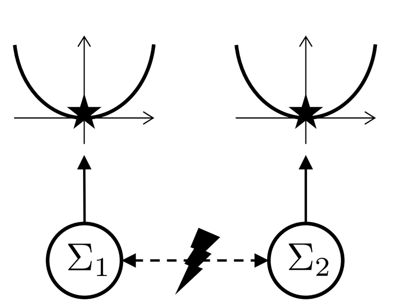
Non-Cooperative Distributed MPC

- Input-coupled linear systems $\Sigma_i: \ x_{t+1}^{[i]} = A_i x_t^{[i]} + B_{ii} u_t^{[i]} + \sum_{j \in \mathcal{N}_i} B_{ij} u_t^{[j]}$

  Running Example: Heavy duty vehicle platooning



- Non-cooperative control strategy
- Each agent optimizes its own performance index
- Neighbor-to-neighbor communication
  - ➢ Once per each sampling time
  - ➢ Transmission is delayed by d time steps



- Distributed MPC (DMPC) with non-cooperative agents

$$\min_{\bar{\mathbf{X}}_t^{[i]}, \bar{\mathbf{V}}_t^{[t]}} J_{t \to t+N}^{[i]}(\bar{\mathbf{V}}_t)$$

$$\text{s.t. } \bar{x}_t^{[i]} = x_t^{[i]} \text{ and } \forall k \in \mathcal{I}_{t:t+N-1}:$$

$$\bar{x}_{k+1}^{[i]} = \Phi_i \bar{x}_k^{[i]} + B_{ii} \bar{v}_k^{[i]} + B_{ij} \tilde{u}_k^{[j]}$$

$$\bar{x}_{k+1}^{[i]} \in \mathbb{X}^{[i]} \ominus \mathbb{W}_{k+1}^{[i]}(d)$$

$$\bar{u}_{k+1}^{[i]} \in \mathbb{U}^{[i]} \ominus K_i \mathbb{W}_{k+1}^{[i]}(d)$$

$$G_i(d)(\bar{v}_k^{[i]} - \bar{v}_{k|t-1}^{[i]}) \le g_i(d)$$

- Input parametrization
  $\bar{u}_k^{[i]} = \bar{v}_k^{[i]} + K_i \bar{x}_k^{[i]}$
  $\Phi_i = A_i + B_{ii} K_i$
- Neighbors' transmitted inputs
  $\tilde{\mathbf{U}}_t^{[j]} = \left[\bar{u}_{t|t}^{[j]}, \ldots, \bar{u}_{t+N-1|t}^{[j]}\right]$

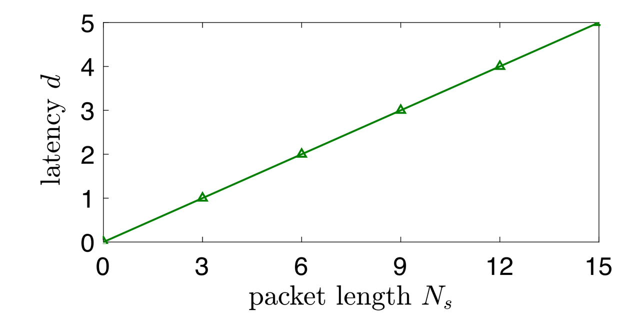
- Latency-dependent constraint tightening that ensures safety

- "Commitment" not to deviate too far away from previously transmitted inputs

Challenge 1: Design the deviation constraints
Challenge 2: Synthesize sets $\mathbb{W}_k^{[i]}$
} Such that the system is safe under latency d

Tuning Communication Latency

- Latency d modeled as a **linear** function of the length of the transmitted packets N_s
- Adverse channel conditions (e.g. deep fades)
- Multiple access channel (e.g. CSMA)
- Tuning parameter: N_s (determines d)

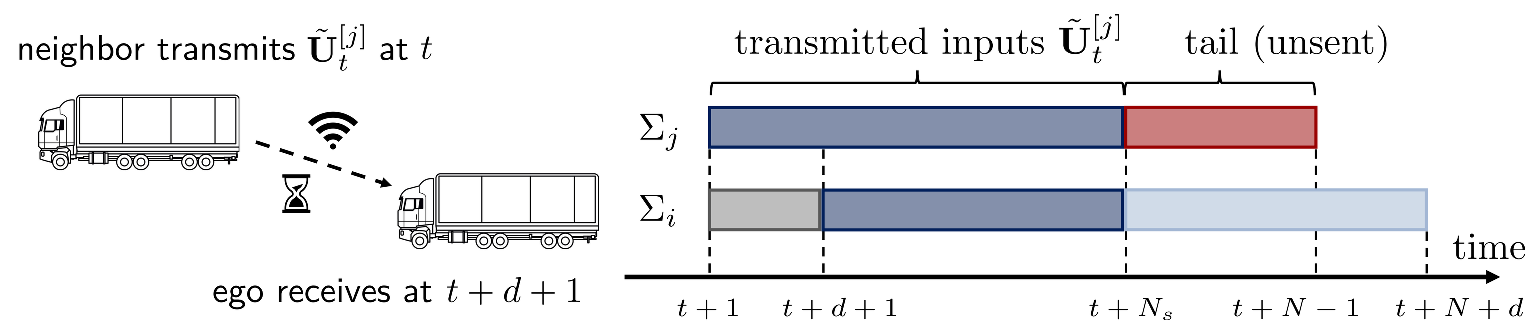


Transmission rules:
- Send first N_s elements in the input sequence computed by MPC at time t, i.e.

$$\tilde{\mathbf{U}}_t^{[j]} = \left[\bar{u}_{t+1|t}^{[j]}, \ldots, \bar{u}_{t+N_s|t}^{[j]}\right]$$

Receiving rules:
- Only $N_s - d$ elements useful by the time $\tilde{\mathbf{U}}_t^{[j]}$ is received
- Estimate the remaining elements ("tail inputs") using neighbor's dynamics:

$$\tilde{u}_k^{[j]} = K_j \tilde{x}_k^{[j]}, \quad \tilde{x}_{k+1}^{[j]} = \Phi_j \tilde{x}_k^{[j]}$$



Latency-Based DMPC Design

- Two sources of uncertainties in decision making of Σ_i:
- Mismatch between $\tilde{\mathbf{U}}_t^{[j]}$ (transmitted) and $\bar{\mathbf{U}}_{t+d+1}^{[j]}$ (currently computed)
- Estimation error of the tail inputs

Deviation constraints (Challenge 1):

$$\bar{v}_{t+k|t}^{[i]} - \bar{v}_{t+k|t-1}^{[i]} \in \mathbb{V}^{[i]}, \quad \forall k = 0, \ldots, N_s - 1$$

$$\bar{v}_{t+k|t}^{[i]} - 0 \in \mathbb{T}^{[i]}, \quad \text{For all tail inputs}$$

Constraint tightening (Challenge 2):
- Error dynamics: $w_{t+k+1}^{[i]} = \Phi_i w_{t+k}^{[i]} + B_{ij}(u_{t+k|t}^{[j]} - \tilde{u}_{t+k}^{[j]}) \in \mathbb{W}_k^{[i]}$
- Exploit structure of the input: $\bar{u}_k^{[i]} = \bar{v}_k^{[i]} + K_i \bar{x}_k^{[i]}$

Computation of the disturbance sets:

$$\mathbb{W}_{k+1}^{[i]} = \Phi_i \mathbb{W}_k^{[i]} \oplus B_{ij}\left(\hat{\mathbb{V}}_k^{[j]} \oplus K_j \hat{\mathbb{X}}_k^{[j]}\right) =: \hat{\mathbb{U}}_k^{[j]}$$

- Deviation in $\bar{v}_k^{[j]}$

$$\hat{\mathbb{V}}_k^{[j]} = \begin{cases} (d+1)\mathbb{V}^{[j]}, & \forall k \in \mathcal{I}_{1:N_s-1} \\ \mathbb{T}^{[j]} \oplus m_k \mathbb{V}^{[j]}, & \forall k \in \mathcal{I}_{N_s:N_s+d} \\ \mathbb{T}^{[j]}, & \text{tail inputs} \end{cases}$$

- Deviation in $\bar{x}_k^{[i]}$

$$\hat{\mathbb{X}}_{k+1}^{[j]} = \Phi_j \hat{\mathbb{X}}_k^{[j]} \oplus B_{jj} \hat{\mathbb{V}}_k^{[j]} \oplus B_{jj-1} \hat{\mathbb{U}}_k^{[j-1]}$$

Algorithm Set design for DMPC

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Input: d, N_s, system model and sets V^[i], T^[i]
for subsystem index i ← 2 to M do
    Compute sets W_k^[i]
    Pass the sets Û_k^[j] to Σ_{i+1}
end for
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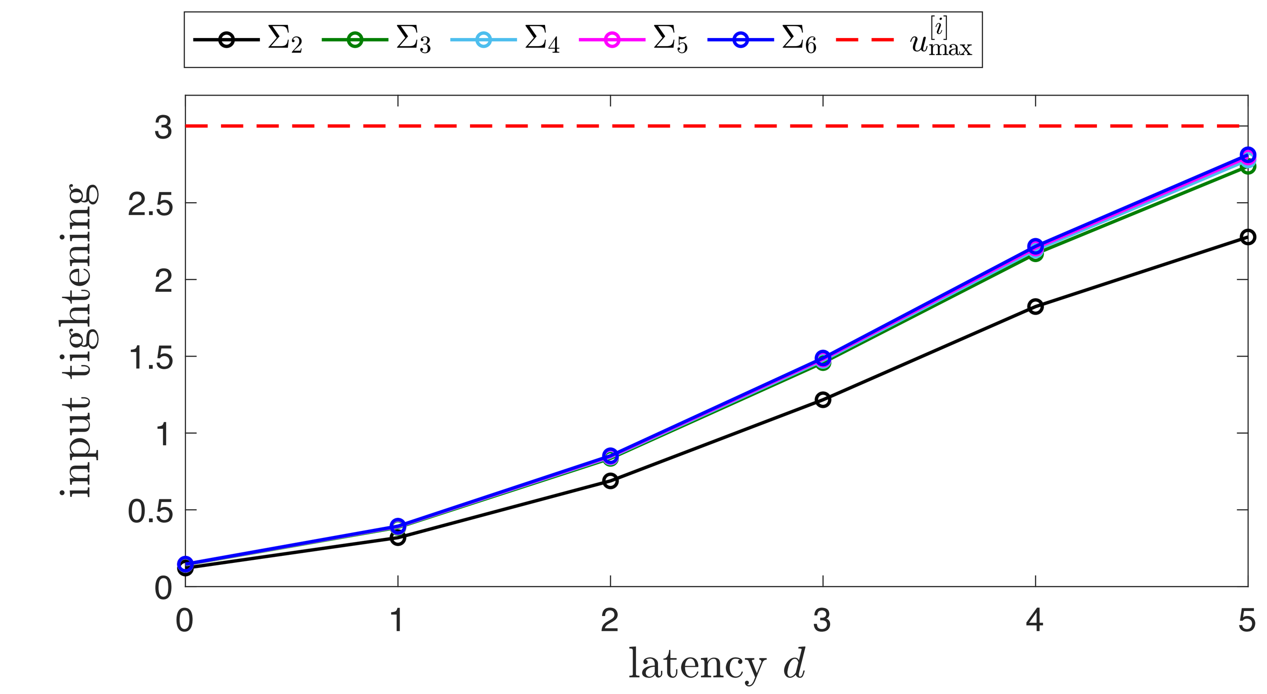
Performance of the DMPC Algorithm

Theorem: For each agent Σ_i, $i = 1, \ldots, M$ in closed loop with the proposed DMPC starting from an initial condition that is feasible, the following holds
1) The controller is recursively feasible for all $t \ge 0$
2) The closed loop system converges asymptotically to the origin

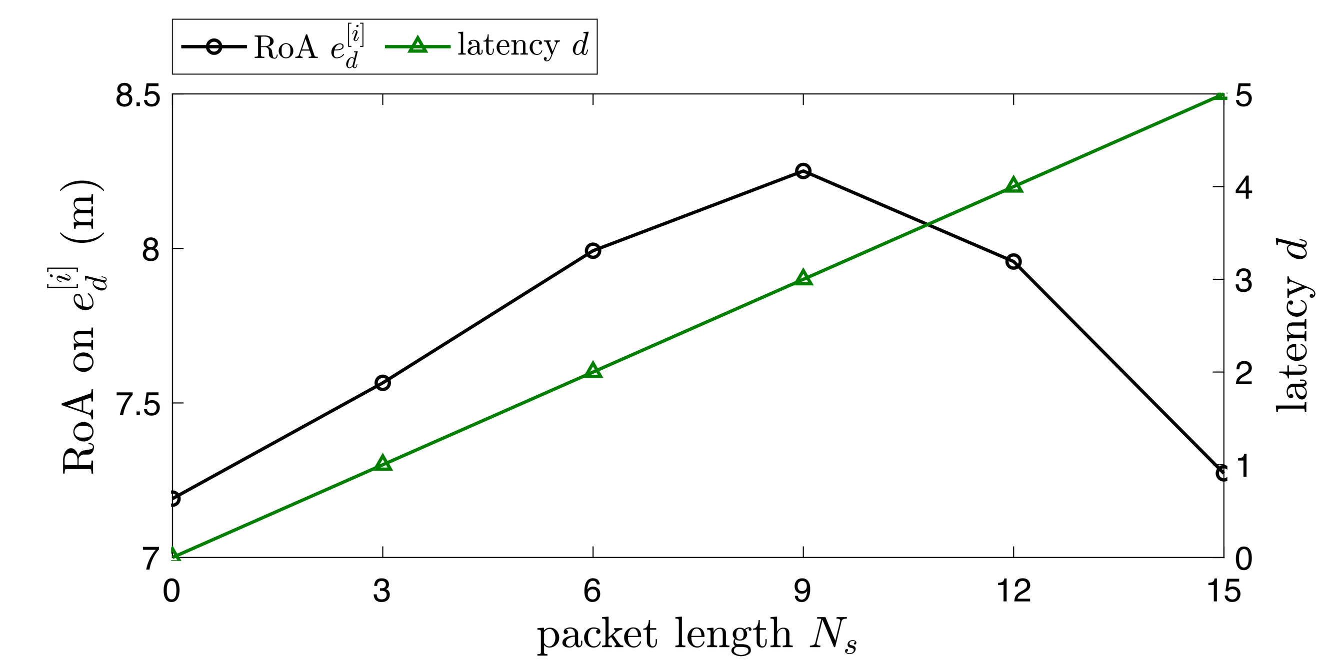
- Proof: Main ideas based on Chisci et al 2001

Numerical Simulations:

- Offline controller design:
- Amount of tightening does not propagate along the chain
- Safety is guaranteed regardless of latency tuning



- Online control: Suitable latency parameter leads to improved performance
- Enlarged region of attraction (RoA)



- Reduced convergence time

Length N_s	0	3	6	9	12	15
T_c (s)	8.1	7.2	6.3	5.1	6.2	6.5

Convergence time

Future Work

- Broader class of systems and network topologies
- Realistic modeling of communication latency
- Stochastic MPC that exploits the randomness of latency

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